Detecting topological order with frustration

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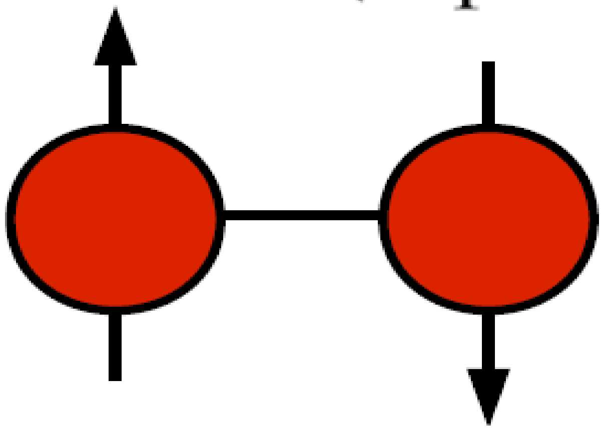
(M. Dalmonte, S. M. Giampaolo, G. Zonzo, P. Zoller, F. I.)

Global many-body physics:
Competition vs. cooperation between many few-body local terms

Interacting Heisenberg spin pair – alone in the universe:

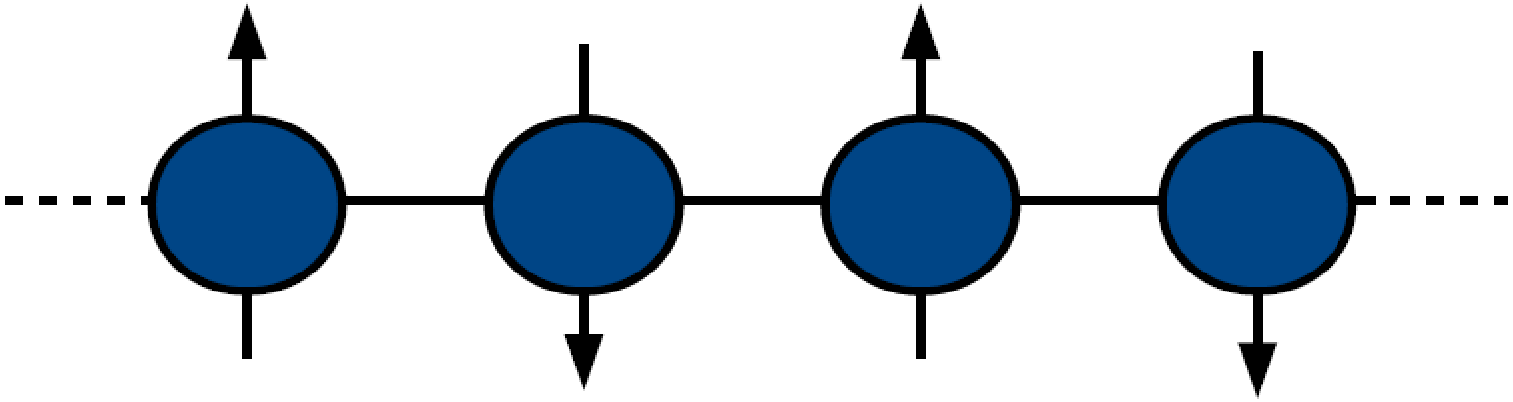
$$H = J(\vec{S}_1 \cdot \vec{S}_2)$$



$$|GS\rangle \equiv |GS_{Local}\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}} \equiv |GS_{Bell}\rangle$$

Many interacting spin pairs: 1-D chain

$$H = \sum J_{ij} \vec{S}_i \cdot \vec{S}_j$$



$$|GS\rangle \equiv |GS_{Global}\rangle$$

Hierarchy of (nonlocal quantum) orders:
Frustration of the local order imposed by the global one

GLIO

Global-to-Local Incompatibility Overlap

- How much does the (projected) global GS differ from the local GS?
- How can we characterize this incompatibility?

$$f = 1 - Tr\{\rho\Pi\}$$

ρ = **Reduced density matrix of the local term – partial trace over the global GS**

$\prod$ = **Local ground state projector – exact GS of the local term**

What does the GLIO actually detect?

- Quantum critical points?
- Nature and classification of the different quantum phases?

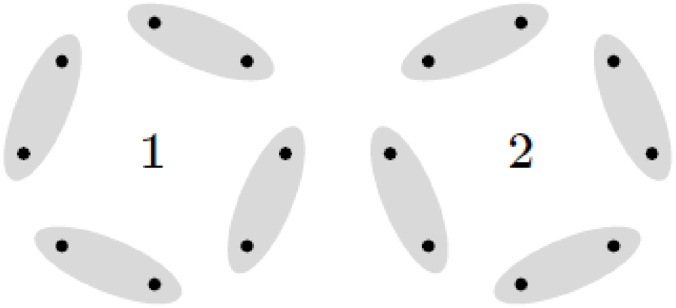
Warm-up (competing interactions): J_1 – J_2 model on the 1-D chain:

$$
\begin{aligned}
H \;=\; & J\cos\phi \sum_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \sin\delta\, S_i^z S_{i+1}^z\right) \\
+\; & J\sin\phi \sum_i \left(S_i^x S_{i+2}^x + S_i^y S_{i+2}^y + \sin\delta\, S_i^z S_{i+2}^z\right)
\end{aligned}
$$

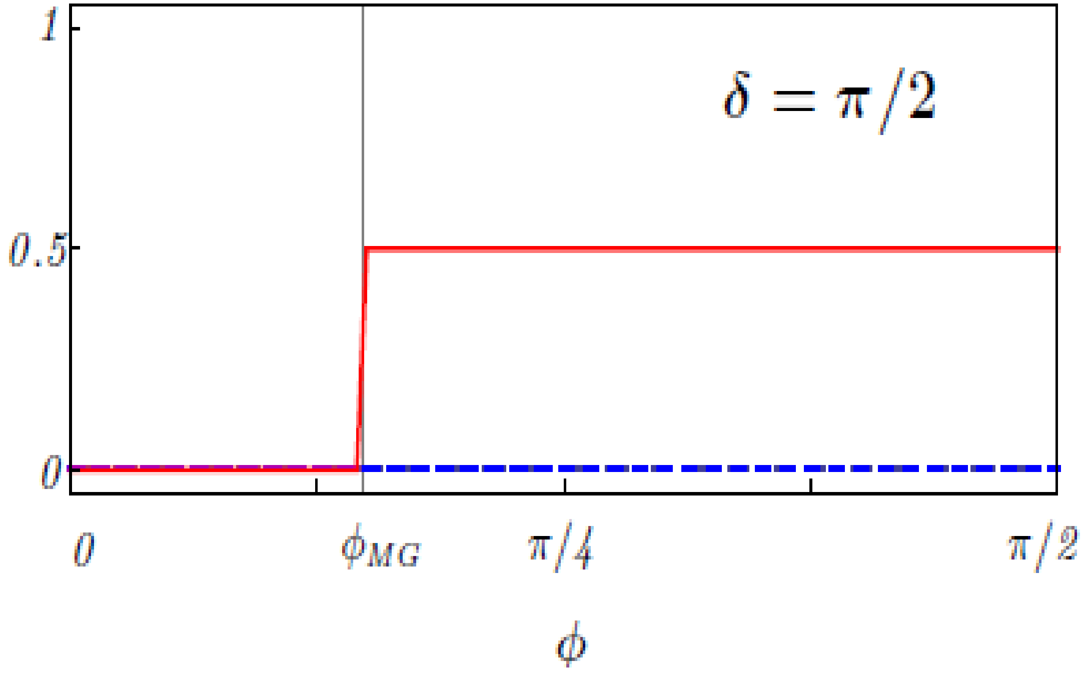
Majumdar-Ghosh point: J_2 = 0.5 J_1 - Exact ground state dimerization

What do we expect? NN frustration = 0 ; NNN frustration strongly enhanced

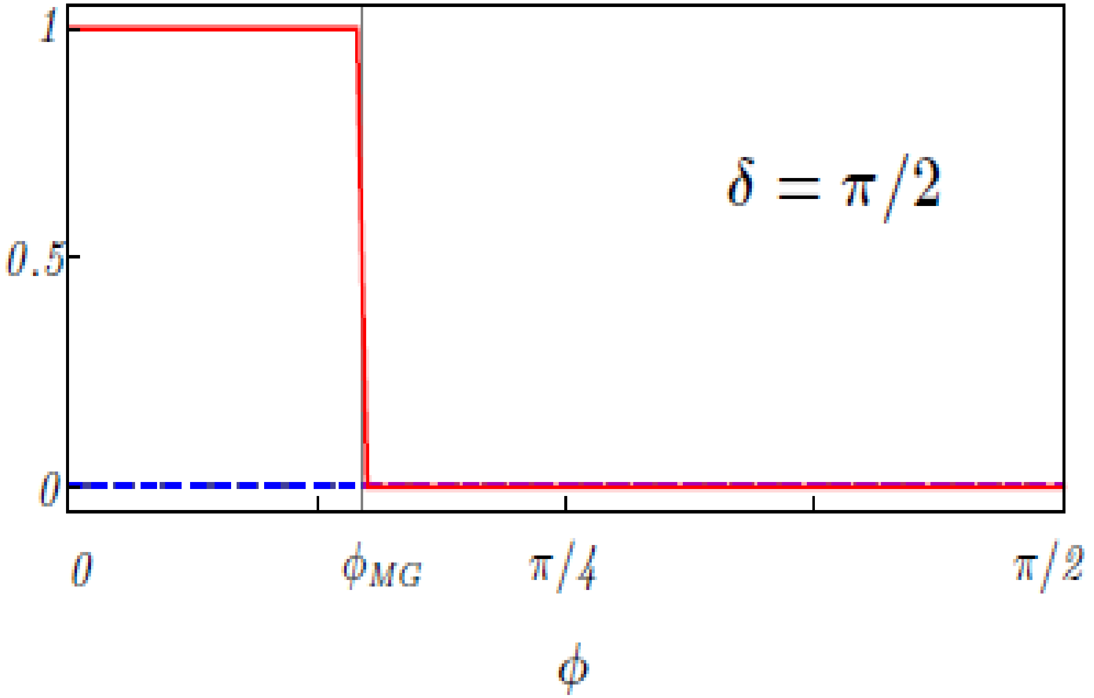
Dimerization (two-fold degenerate GS):



NN GLIO (NN frustration):



NNN GLIO (NNN frustration):



Scaling with the number of local terms:

1) detecting transitions

2) classifying the nonlocal strength of the quantum order

Symmetry-protected topological order, Majorana fermions, edge states:

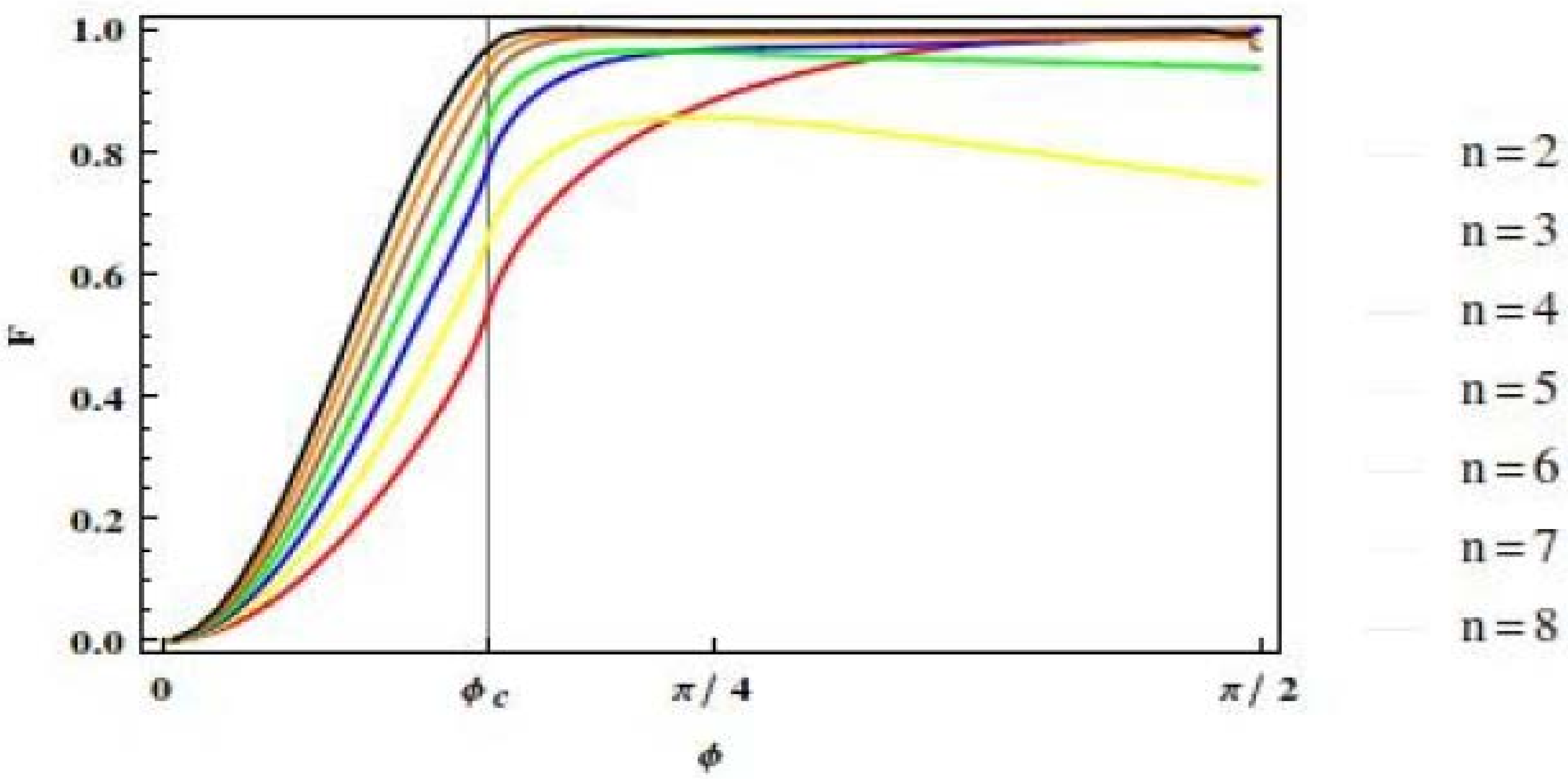
Case A) The 1-D Kitaev chain: fully incompatible (non-commuting) local interaction terms

$$H = -\sin(\phi)\sum_j (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) - \cos(\phi)\sum_j (c_j^\dagger c_j - \frac{1}{2})$$

$$+\sin(\phi)\sum_j c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger$$

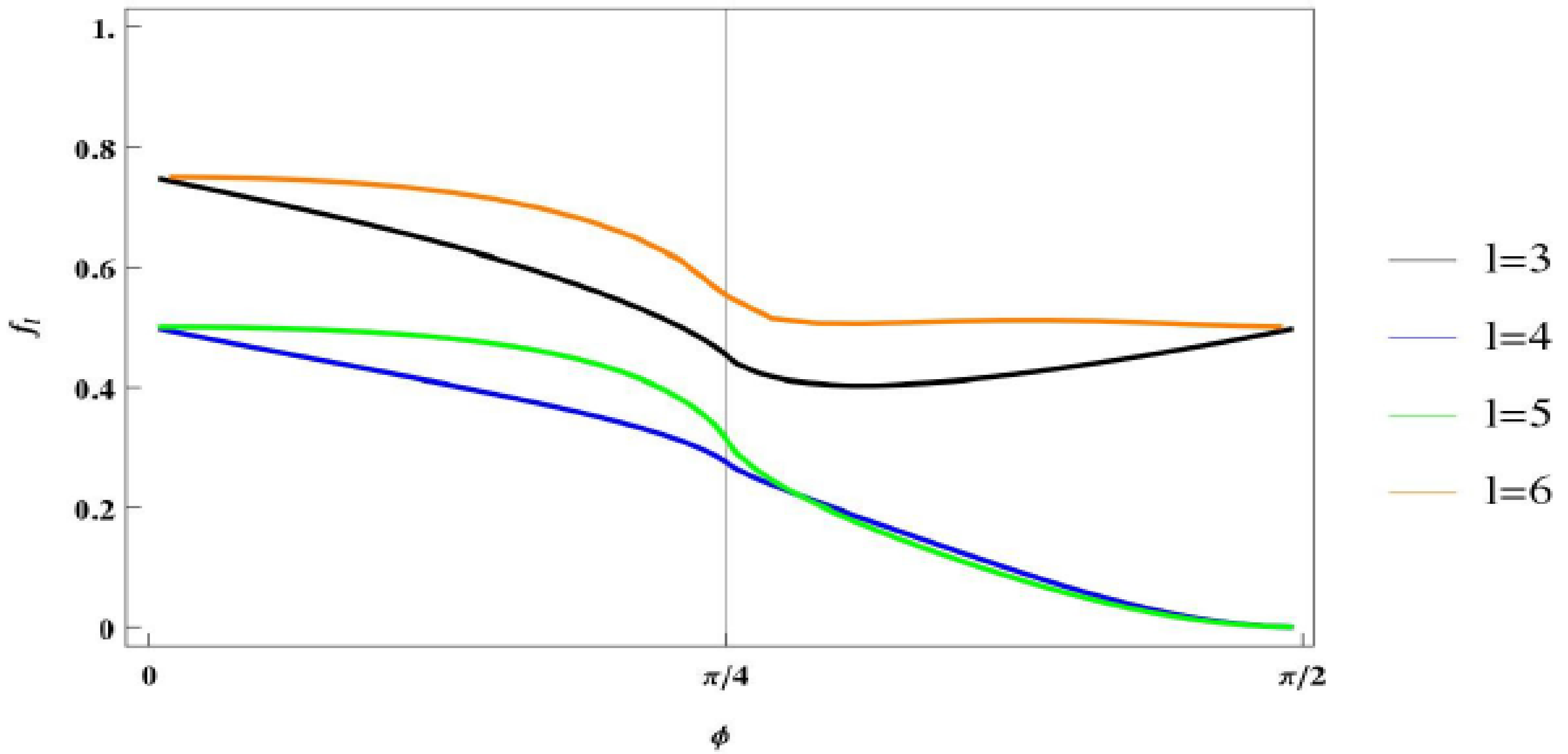
Complete topological degeneracy under Z_2 symmetry.

Deepest topological phase ↔ saturation of frustration (maximum of the GLIO, maximum possible incompatibility between local and global order).



Case B) The 1-D cluster-Ising chain: topological order with partially compatible local interactions

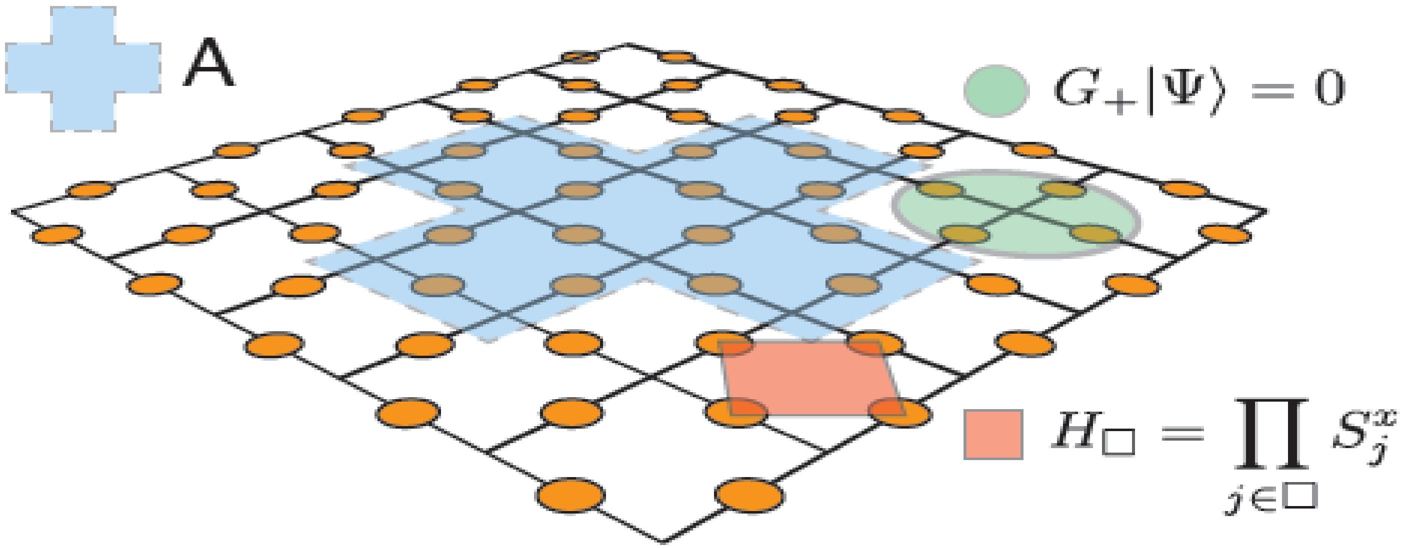
$$H = -\cos(\phi)\sum_j \sigma_j^x \sigma_{j+1}^z \sigma_{j+2}^x + \sin(\phi)\sum_j \sigma_j^y \sigma_{j+1}^y$$



Case C) The 2-D toric code and Ising lattice gauge theory: topological order with compatible local interactions plus global gauge constraint on the physical states

$$H = -\sum_{\square} H_{\square} + h \sum_{j} S_j^z \qquad H_{\square} = \prod_{j \in +} S_j^x$$

Gauge constraint (Gauss law) on physical states: $G_+ |\Psi\rangle_{ph} = 0, \quad G_+ = \prod_{j \in +} S_j^z - 1$



Exactly solvable points:

h = 0   (fully deconfined, topological phase)

h = ∞   (fully confined, paramagnetic phase)

Results:

I) Confined phase: $f_{\mathrm{conf}} = 0$

II) Topological phase: $f_{\mathrm{deconf}} = 1 - 2/p$

Where *p = perimeter of the local cross.*

As p → ∞ , $f_{\mathrm{deconf}} \rightarrow 0$

Conclusions and outlook

- The GLIO is a universal measure of frustration
- Detects transitions and classifies different quantum orders according to their degree of nonlocality (mutual incompatibility between local interactions, and/or global constraints). Hierarchy of quantum orders.
- A way towards axiomatic measures of quantum nonlocality?
- Defined for any system and in all dimensions (topological entropy not defined in 1D)
- Overlap, simple nonlinear functional of density matrices: measurable in Hong-Ou-Mandel type experiments (seminal work by Ekert)
- With ultracold atoms in optical lattices (Zoller theory, Greiner, first experiments on atomic beam splitters)
- With ions? Other systems?

Path to experiments

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Direct Estimations of Linear and Nonlinear Functionals of a Quantum State

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We present a simple quantum network, based on the controlled-SWAP gate, that can extract certain properties of quantum states without recourse to quantum tomography. It can be used as a basic building block for direct quantum estimations of both linear and nonlinear functionals of any density operator. The network has many potential applications ranging from purity tests and eigenvalue estimations to direct characterization of some properties of quantum channels. Experimental realizations of the proposed network are within the reach of quantum technology that is currently being developed.

$$\rho = \varrho_a \otimes \varrho_b \qquad\qquad \mathrm{Tr}\rho U = \mathrm{v}e^{i\alpha}, \qquad (1)$$

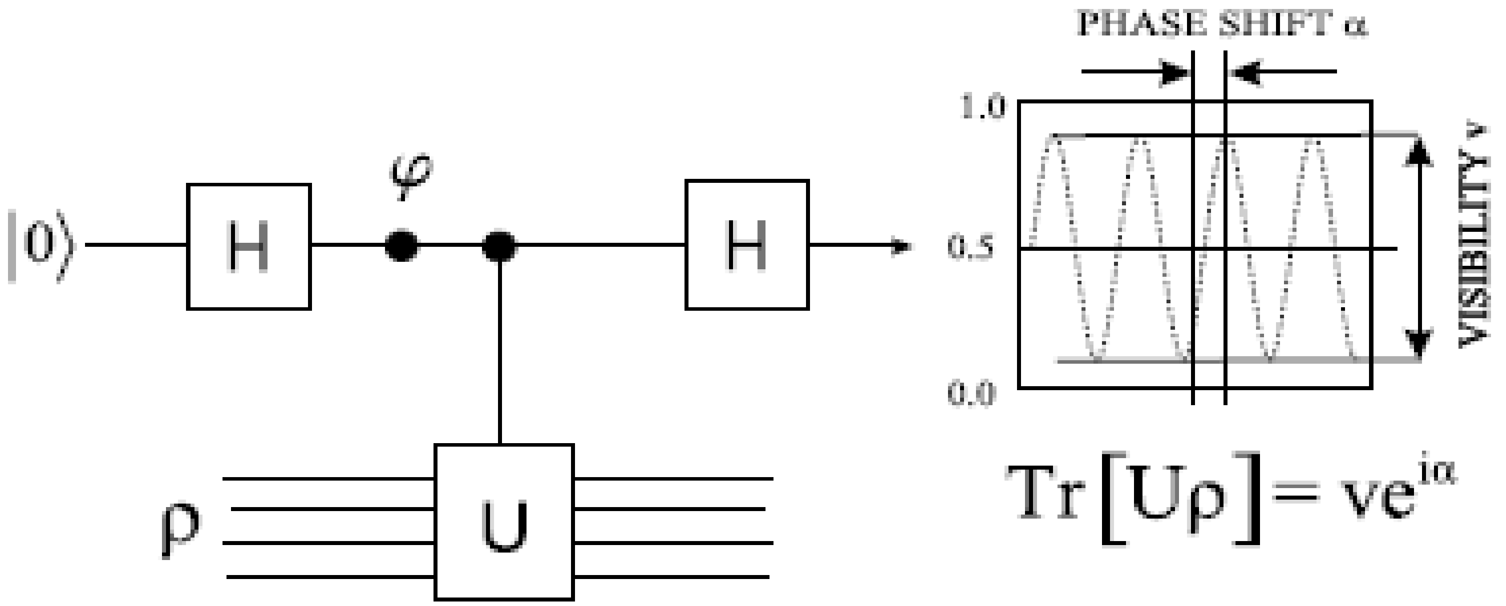

FIG. 1. Both the visibility and the shift of the interference patterns of a single qubit (top line) are affected by the controlled-U operation on a general state, ρ.

$$\mathrm{v} = \mathrm{Tr}V(\varrho_a \otimes \varrho_b) = \mathrm{Tr}\varrho_a\varrho_b$$

- V is the swap operator: V|α>|β> = |β>|α>

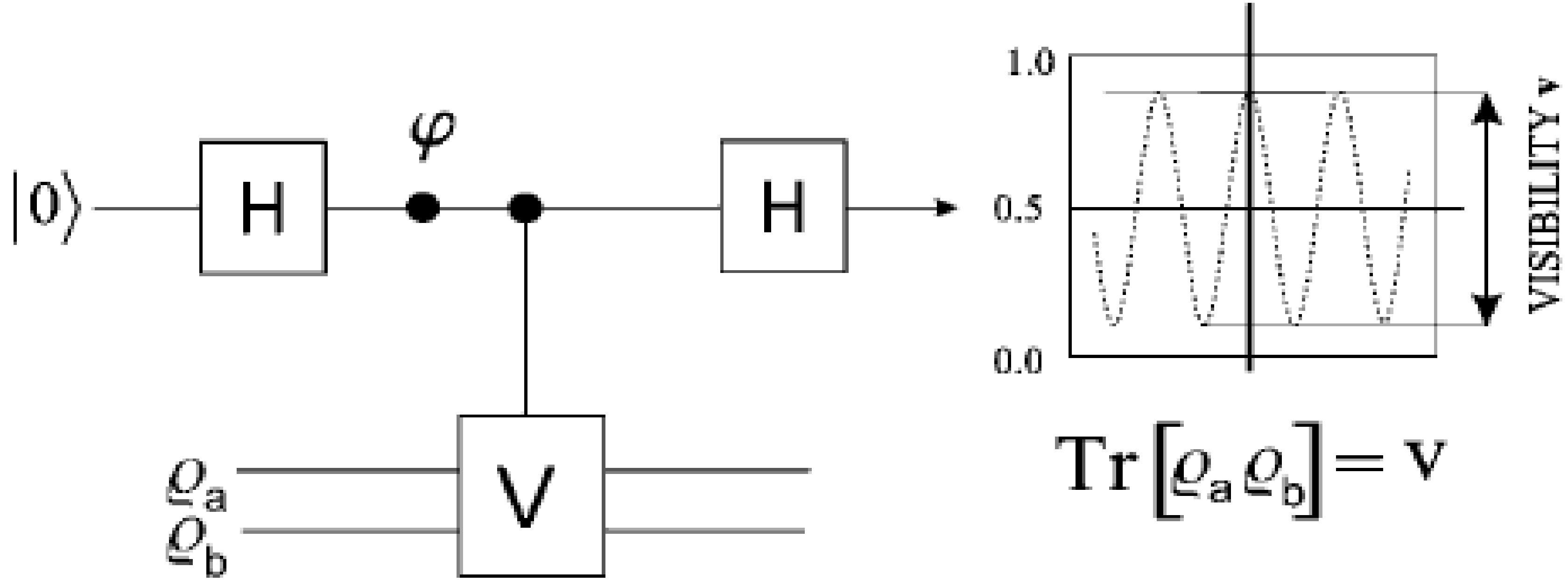

FIG. 2. A quantum network for direct estimations of both linear and nonlinear functions of state. The probability of finding the control (top line) qubit in state $|0\rangle$ at the output depends on the overlap of the two target states (two bottom lines). Thus estimation of this probability leads directly to an estimation of $\mathrm{Tr}\varrho_a\varrho_b = \mathrm{v} = 2P_0 - 1$.

Measuring Entanglement Growth in Quench Dynamics of Bosons in an Optical Lattice

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We discuss a scheme to measure the many-body entanglement growth during quench dynamics with bosonic atoms in optical lattices. By making use of a 1D or 2D setup in which two copies of the same state are prepared, we show how arbitrary order Rényi entropies can be extracted by using tunnel coupling between the copies and measurement of the parity of on-site occupation numbers, as has been performed in recent experiments. We illustrate these ideas for a superfluid-Mott insulator quench in the Bose-Hubbard model, and also for hard-core bosons, and show that the scheme is robust against imperfections in the measurements.

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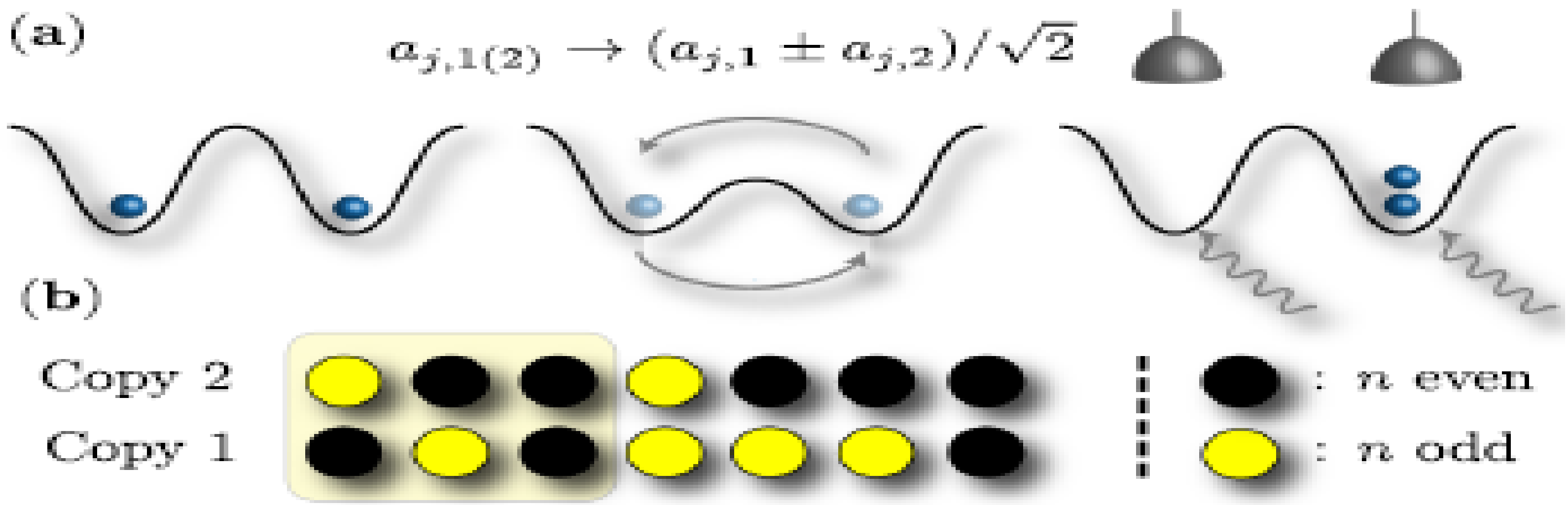

FIG. 1 (color online). (a) Measurement of $n = 2$ Rényi entropy for bosons in an optical lattice. First, two instances of the many-body state are produced (shown here for a single site in a 1D chain or 2D plane). Then tunneling is switched off within each copy, and the barrier between the copies is lowered to realize a beam-splitter operation between the copies. Finally, the parity of the atom number is measured at each site. This measurement is repeated to obtain expectation values for the swap operator V_2, from which the Rényi entropy can be computed (see the text). (b) Example measurement outcome for a single shot on a quantum chain. Here the measurement result for the whole system swap operator $V_2^{\{1,\ldots,7\}}$ is 1, since the total number of particles in copy 2 is even. For the swap of the first three sites $V_2^{\{1,2,3\}}$ is -1, since this number is odd.