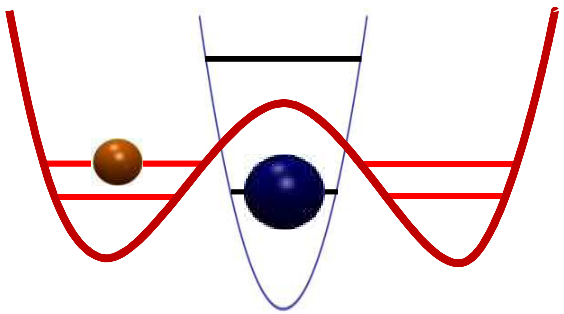
Quantum simulation with cold ions and atoms



Ulm 2014

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Ion trapping group

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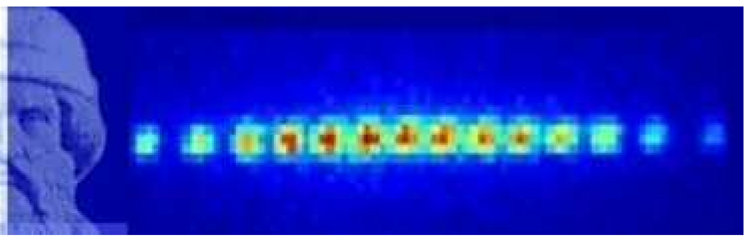

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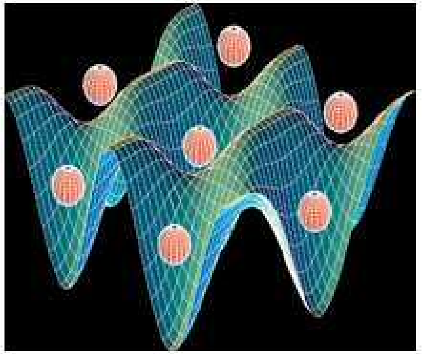
Atoms or ions

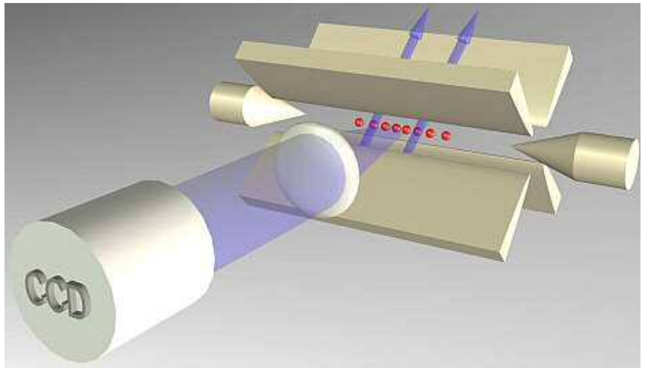
Cold atoms or ion are great for studying quantum many-body physics

Atoms in an optical lattice:
'Artificial solids'



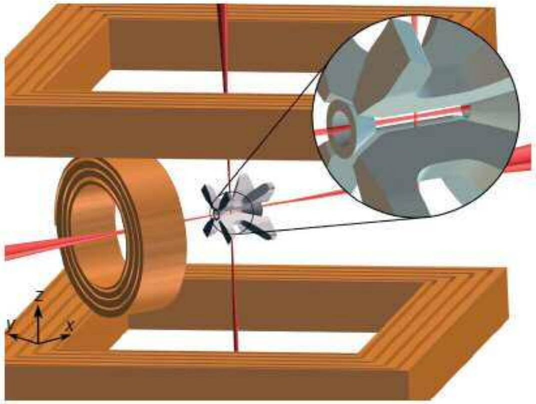
Easily scalable
Fermionic statistics
Harder to get long-range interaction
Harder to control/measure

Trapped ions:
'Arrays of interacting spins'

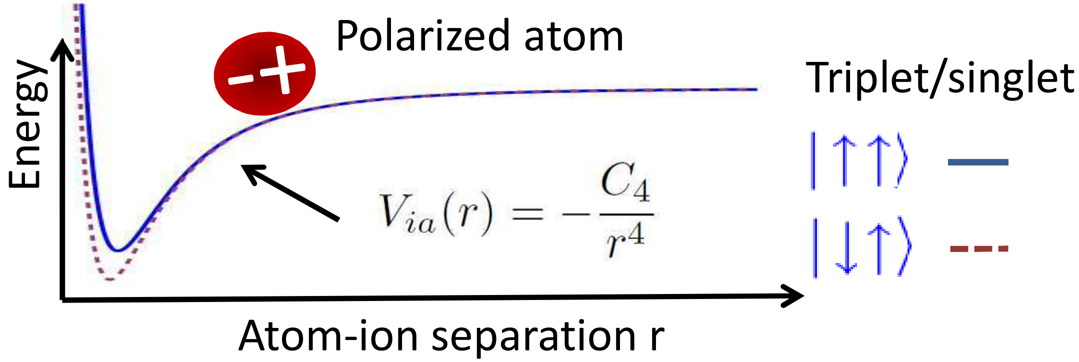


Superb control and readout
Long range interactions
Not easy to scale
No Fermionic statistics

Hybrid atom-ion systems



Picture: Michael Köhl

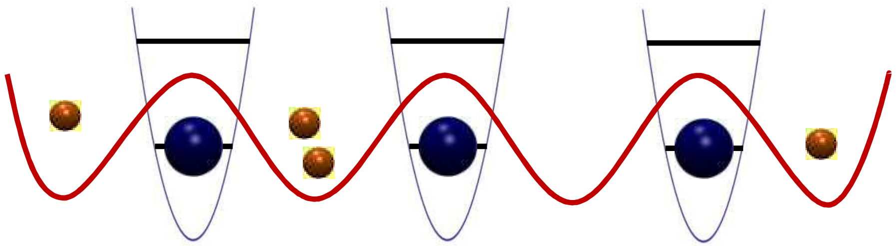


→ Sympathetic cooling
→ Ultracold collisions
→ Cold chemistry

Group	Species	S-wave limit (μK)	Interaction range (nm)	
MIT	^{172}Yb / $^{174}Yb^+$	0.044	252	Bosons
Ulm/Freiburg	^{87}Rb / $^{138}Ba^+$	0.052	295	Bosons
Cambridge	^{87}Rb / $^{174}Yb^+$	0.044	307	Bosons
	^{40}K / $^{174}Yb^+$	0.15	219	Fermions
Mainz	**^{6}Li / $^{174}Yb^+$**	**8.7**	**70**	Fermions

A. T. Grier *et al.*, PRL **102**, 223201 (2009).
C. Zipkes *et al.*, Nature **464**, 388 (2010).
S. Schmid *et al.*, PRL **105**, 133202 (2010).
S. Schmid *et al.*, PRL **105**, 133202 (2010).
A.Härter and J. Hecker Denschlag, arXiv:1309.5799

A solid look-alike?

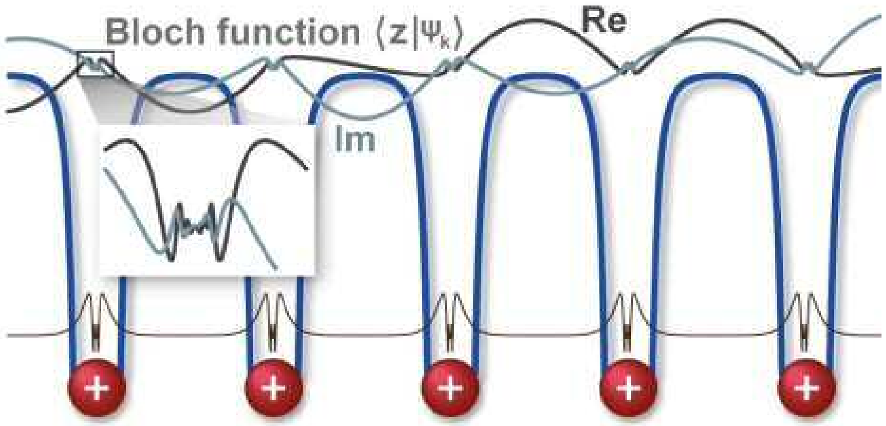


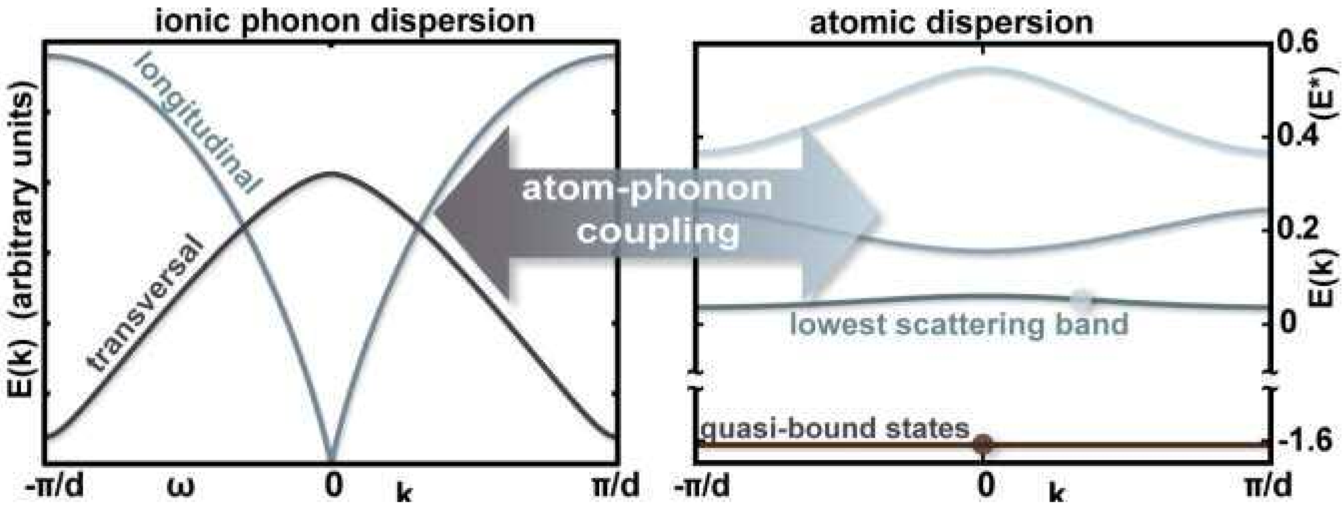
→ A string of ions overlapped with a cloud of ultracold fermions: an artificial solid??

	Solid state	^{6}Li-^{174}Yb$^+$	^{40}K-^{40}Ca$^+$
Lattice spacing d (nm)	0.3–0.6	10^3–10^4	10^3–10^4
Length scale R^* (nm)	0.026	71	245
Energy scale E^* (kHz)	10^{13}	166	2.1
d/R^*	10–20	14–140	4–40
m_i/m_f	10^4–10^5	29	1.0
Fermi energy (MHz)	10^8	0.02	0.02
Phonon energy (MHz)	10^6	0.01–10	0.01–10

Bandstructure

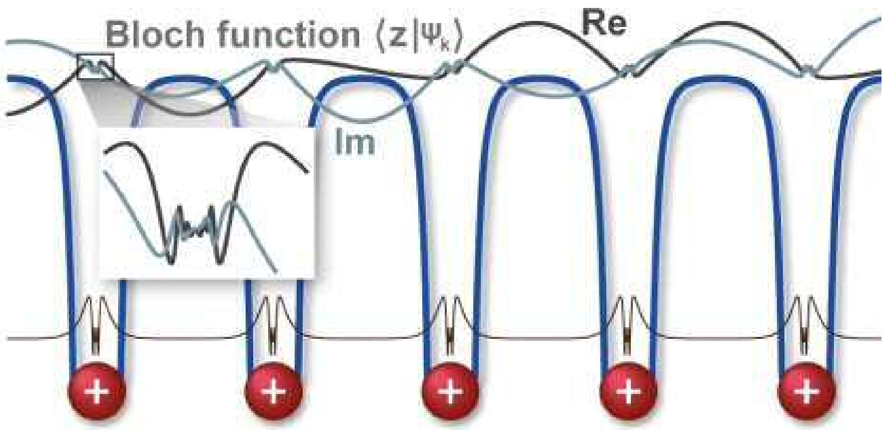
→ Combine quantum defect theory with the Bloch theorem
→ Find phonons of ion string
→ Compute first order coupling





U. Bissbort, D. Cocks, A. Negretti, Z. Idziaszek, T. Calarco, W. Hofstetter, F. Schmidt-Kaler and RG, Phys. Rev. Lett. **111**, 080501 (2013).

Fermion-phonon coupling



Ion crystal + atoms: Fröhlich model

- **Atomic bandstructure**
- **Fermion-phonon coupling**
- **phonon mediated interactions**
- **Polarons**
- **Peierls instabilities**

Fröhlich type Hamiltonian

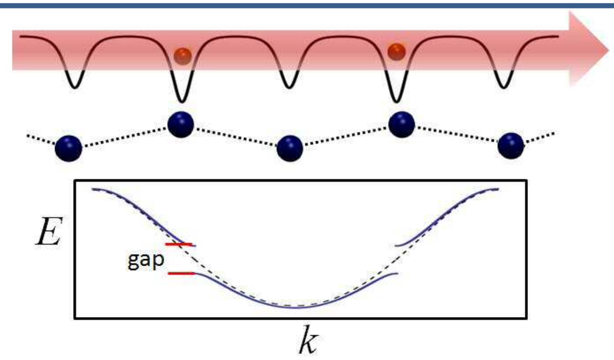
$$\mathcal{H} = \underbrace{\sum_n \hbar\omega_n a_n^\dagger a_n}_{\text{Phonons}} + \underbrace{\sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}}^\dagger c_{\mathbf{k}}}_{\text{Fermions}} + \underbrace{\sum_{\mathbf{kk}'s} \lambda_{\mathbf{kk}'s}(a_s^\dagger + a_s) c_{\mathbf{k}}^\dagger c_{\mathbf{k}'}}_{\text{Fermion-phonon coupling}}$$

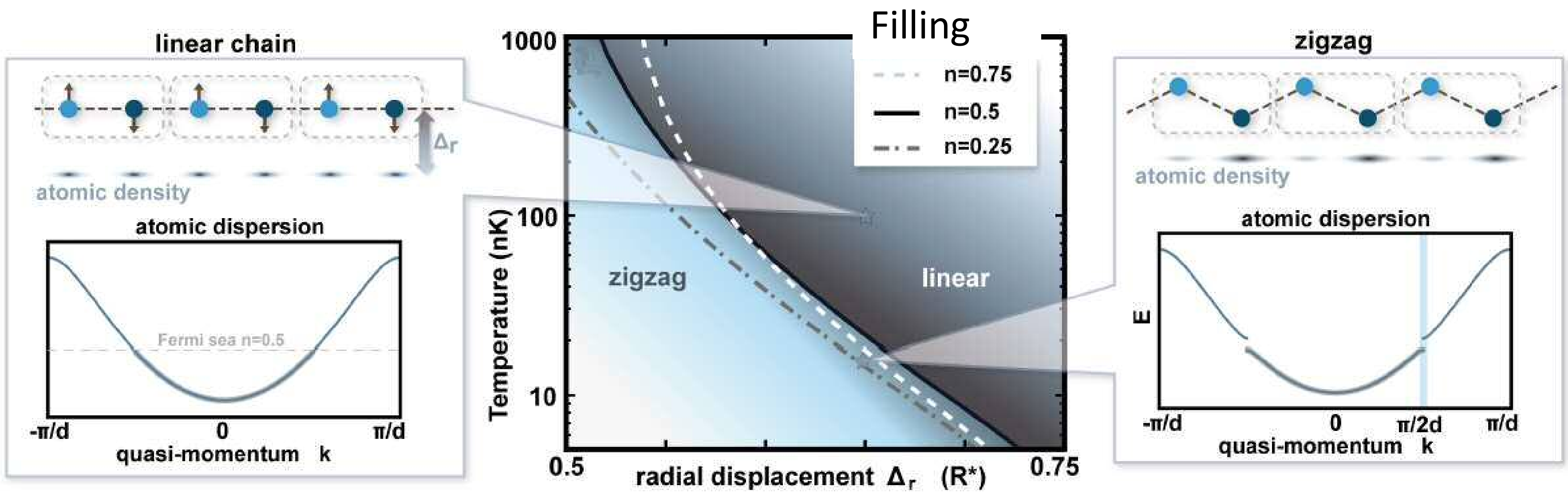
Polarons, Phonon-mediated interactions, Peierls instability,….

Peierls type instability

Peierls transition

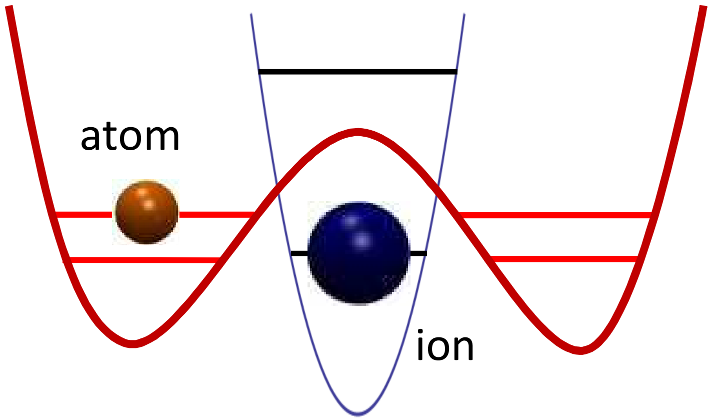
- Conduction to insulator transitions in 1D
- Caused here by *transverse* phonons
- No ion-atom overlap
- Fermionic effect!
- Here: $^{40}Ca^{+}$ and ^{40}K





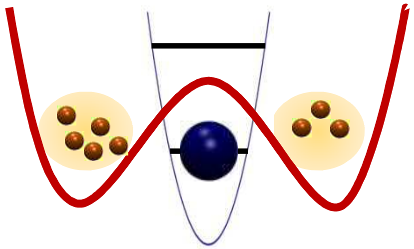
A closer look at the 'unit cell'

A single atom in a double-well potential with a single trapped ion in its center.

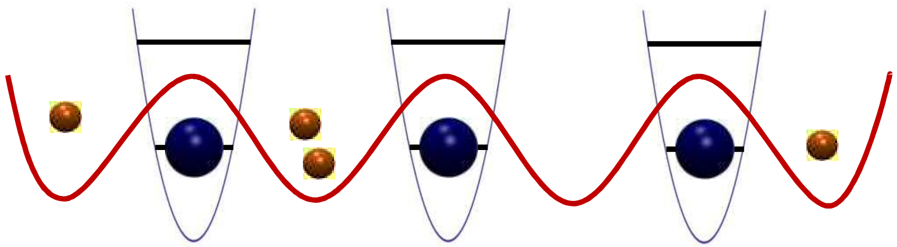


Why this system?

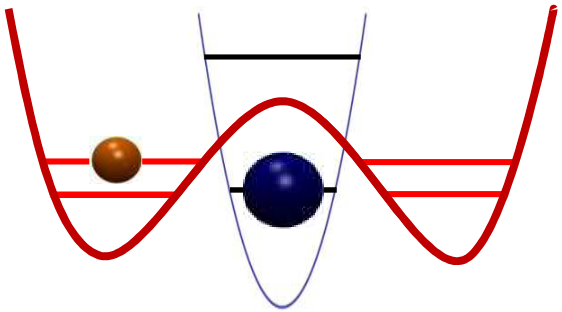
- Because it is not too easy, not too hard
- Many atoms: Atomic Josephson Junction
- Add spin degree of freedom
- Easy to build up larger systems from unit cell



RG, Negretti *et al., PRL* **109**, 080402 (2012).
J. Joger et al., *arxiv:1404.1223*



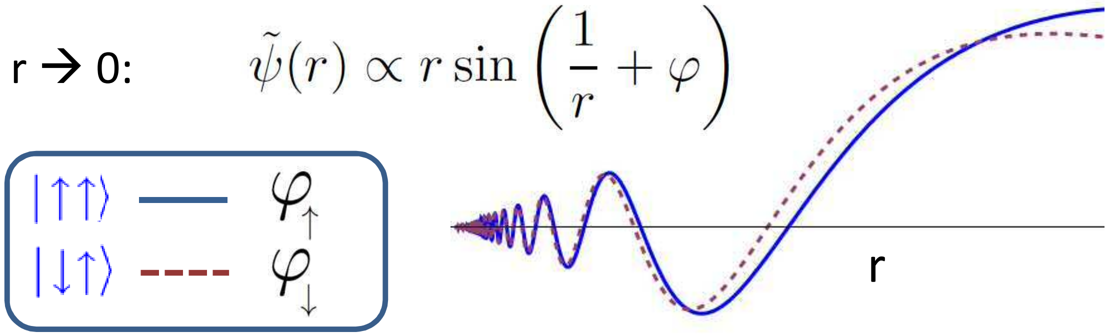
Dealing with the ion: Quantum defect theory



→ Atom-ion interaction at long range

$$V_{ia}(r) = -\frac{C_4}{r^4}$$

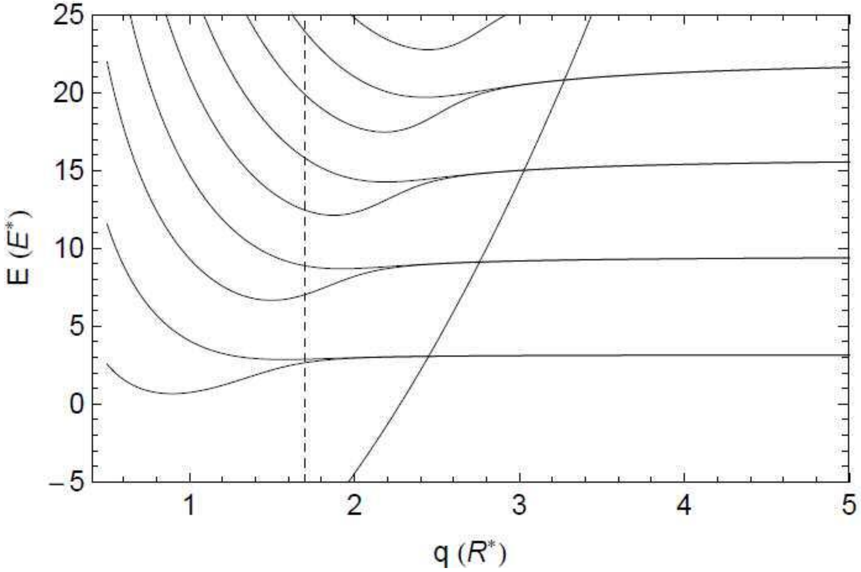
→ Wave function for r → 0: $\tilde{\psi}(r) \propto r \sin\left(\frac{1}{r} + \varphi\right)$

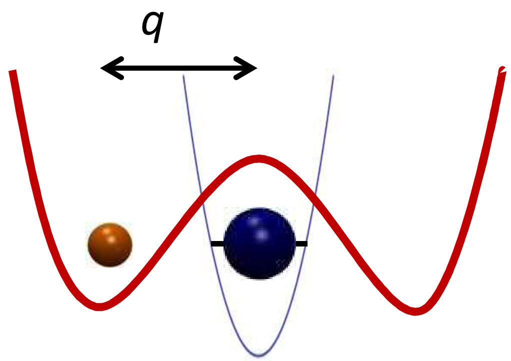


→ φ is related to the s-wave scattering length: $a = -R^* \cot\varphi$

Z. Idziazsek, T. Calarco and P. Zoller, PRA **76**, 033409 (2007).

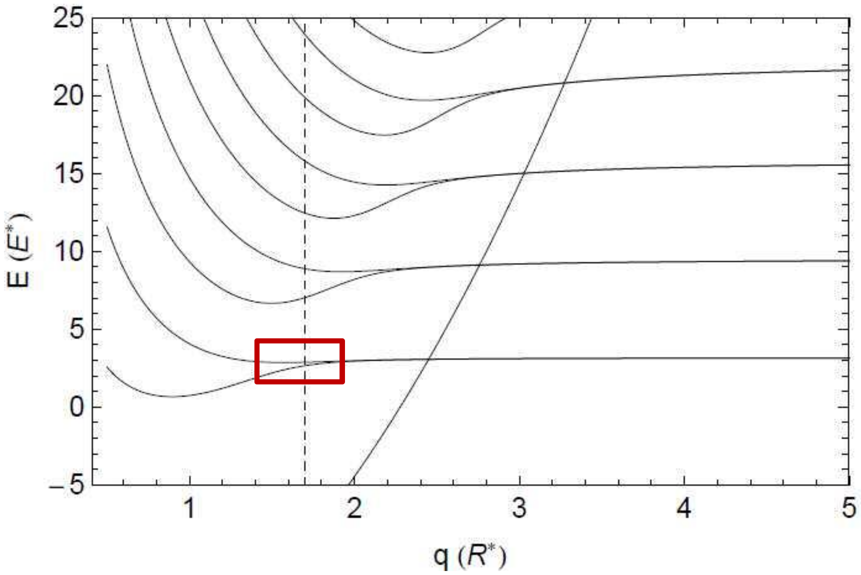
The simplest model: A static ion

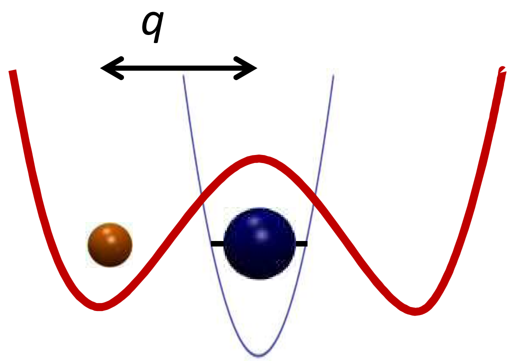




$$H_a = \frac{p_a^2}{2m_a} + V_{dw}(r_a) - \frac{C_4}{r_a^4}$$

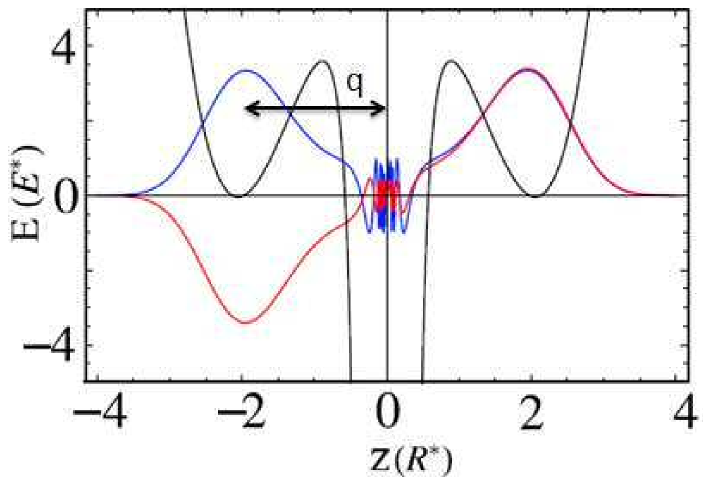
The simplest model: A static ion





$$H_a = \frac{p_a^2}{2m_a} + V_{dw}(r_a) - \frac{C_4}{r_a^4}$$

'Ground state' orbitals



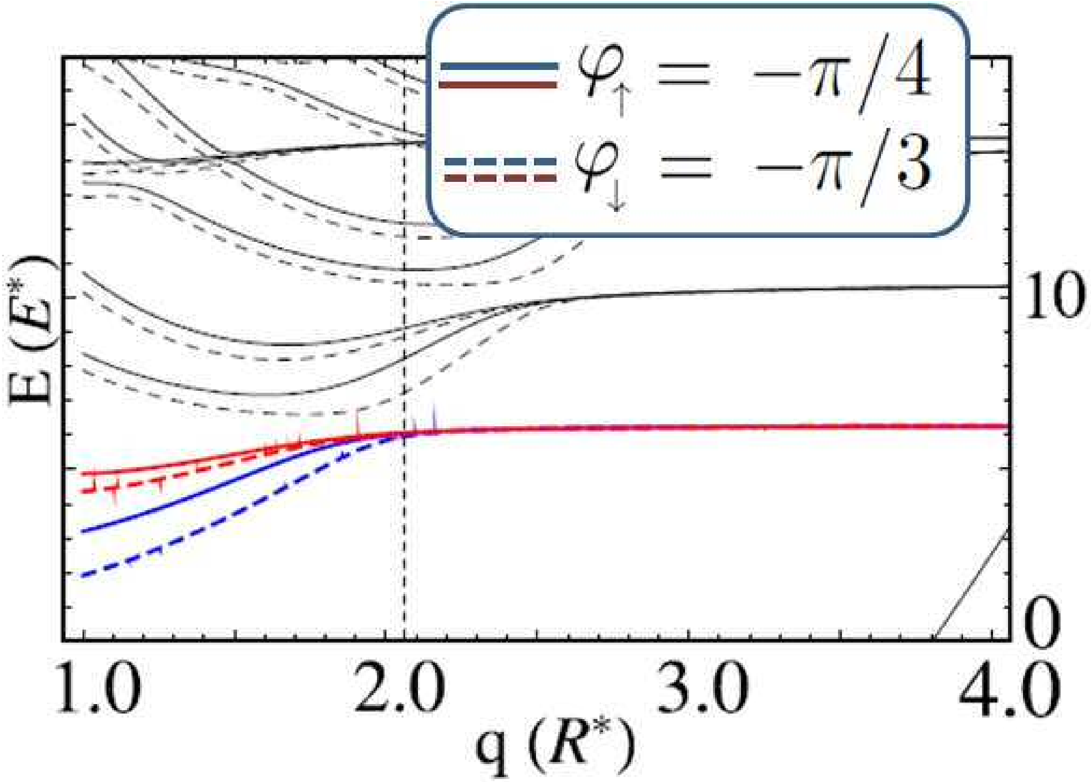
→ Large inter well distance: independent wells, No coupling to ion.

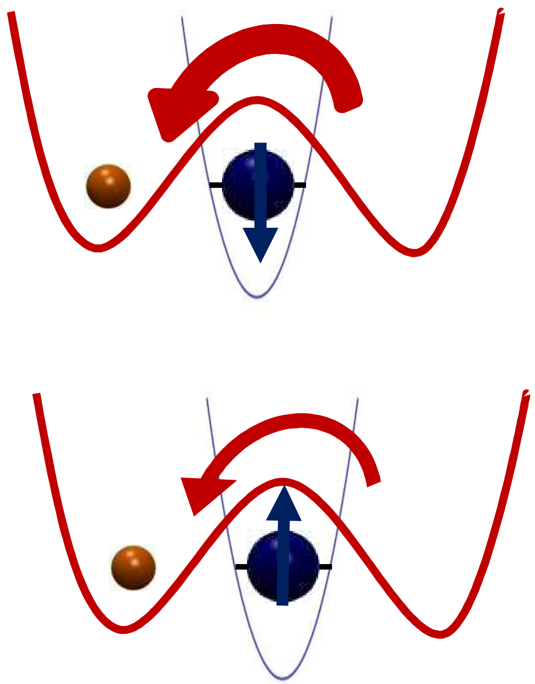
→ Closer distances: splitting into even and odd states.

→ 2-mode approximation!

→ Small coupling to molecular states.

Spin dependence





Ground states are superpositions of localised modes

$$\Phi_{g,e}(\mathbf{r}) = (\Phi_L(\mathbf{r}) \pm \Phi_R(\mathbf{r}))/\sqrt{2}$$

Interwell coupling:

$$\Omega = \Delta E/\hbar$$
$$\Delta E = E_e - E_g$$

Many atoms (bosons)

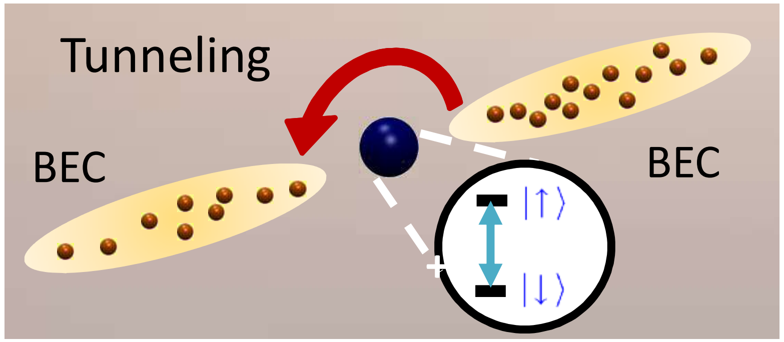
Bose-Hubbard Hamiltonian:

$$\hat{H} = \hbar\hat{J}\left(\hat{c}_L^\dagger\hat{c}_R + \hat{c}_R^\dagger\hat{c}_L\right) + \frac{\hbar}{4}\hat{U}\left(\hat{c}_R^\dagger\hat{c}_R - \hat{c}_L^\dagger\hat{c}_L\right)^2$$

<u>State dependent!</u>

$$\Delta E = E_e - E_g \qquad J = \Delta E/(2\hbar)$$

$$U = \frac{U_0}{\hbar}\int d\mathbf{r}|\Phi_L(\mathbf{r})|^4 \qquad U_0 = \frac{4\pi a_a \hbar^2}{m_a}$$



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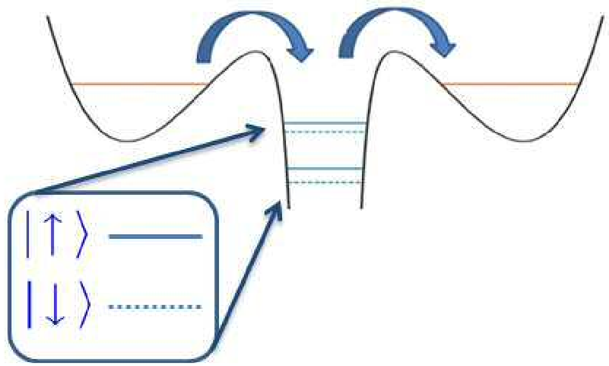
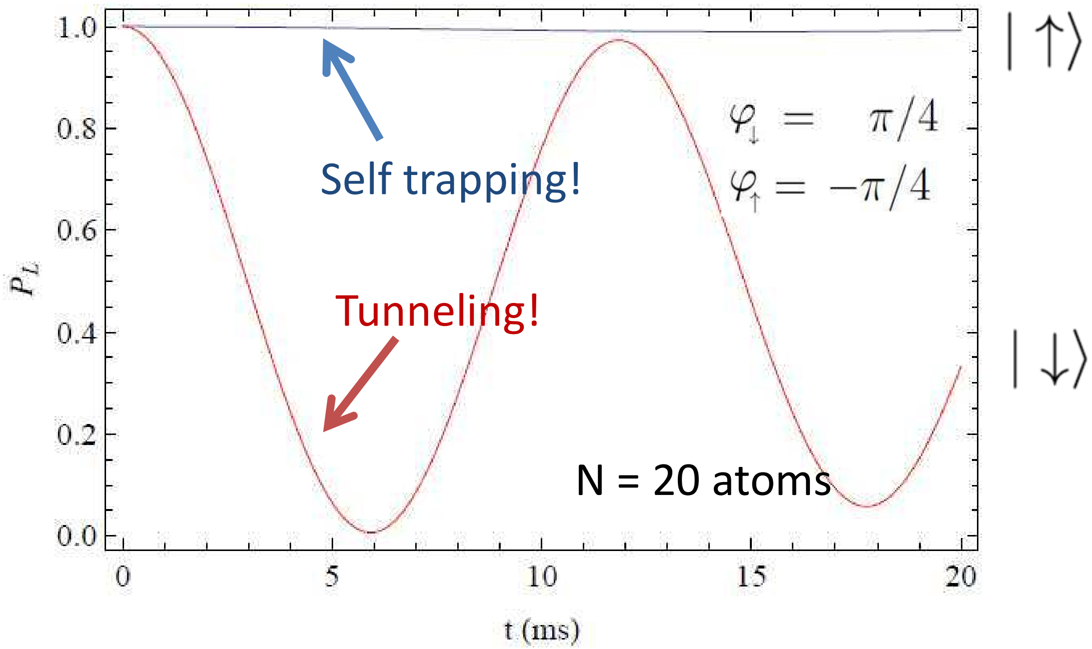
Picture: 3-well



Related work:
U. R. Fischer *et al.,* PRA **77**, 031602R (2008).

Superpositions of many-body dynamics

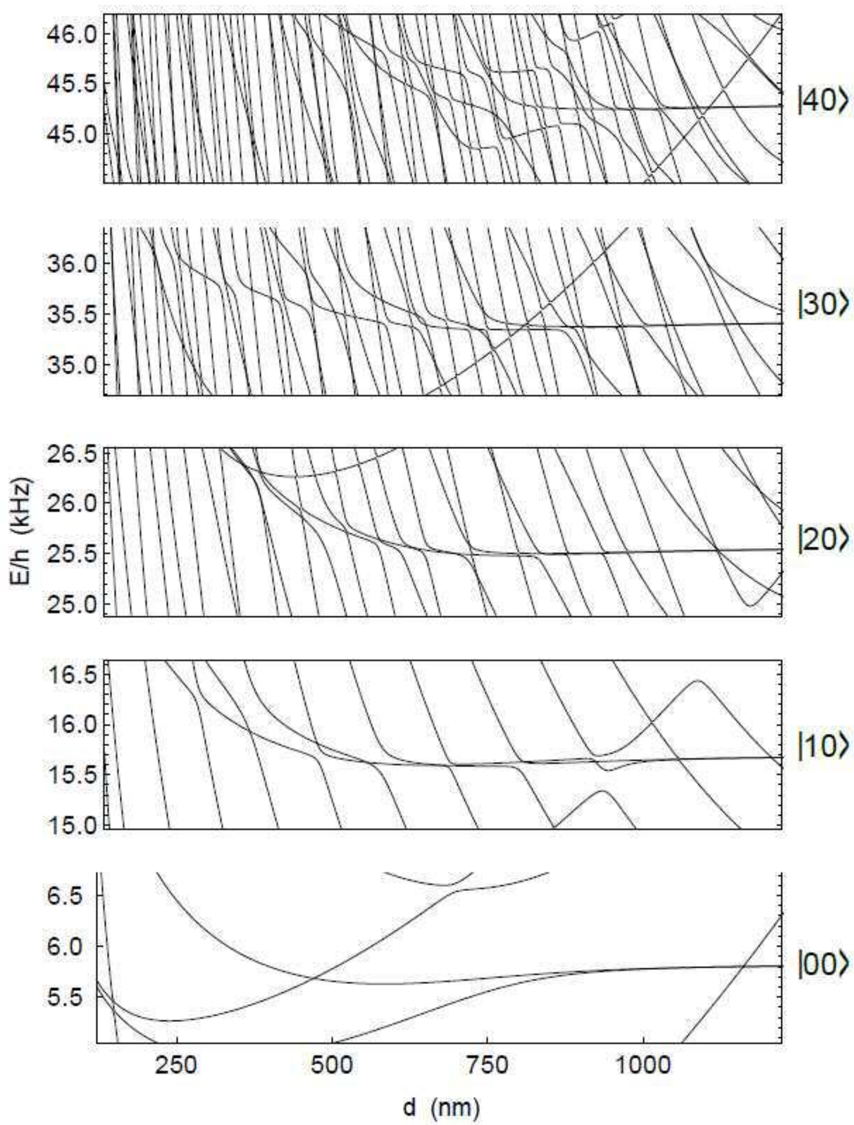


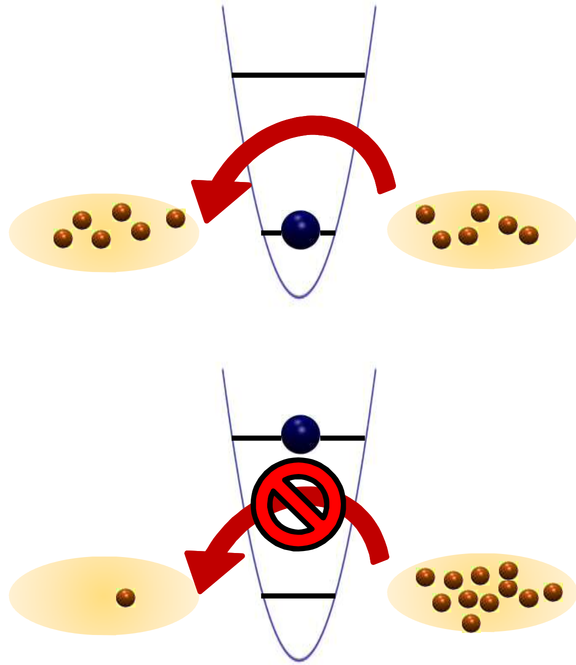
spin

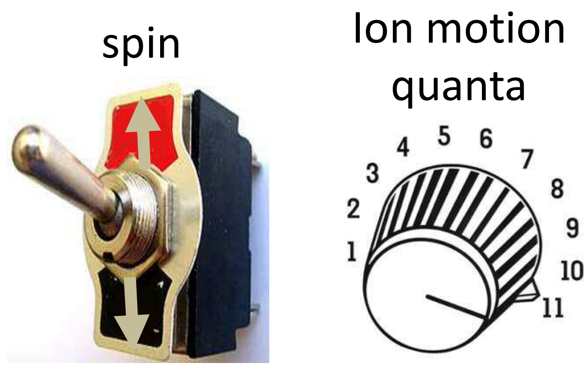
Rb and Yb^+, local trapfreqs of 2π 200 Hz

→ Requires sufficiently large difference between spin short range phases.
→ Need to go beyond BH model for long time scales
K. Sakmann *et al.*, Phys. Rev. Lett. **103**, 220601 (2009).

Controlling tunneling with ion motion







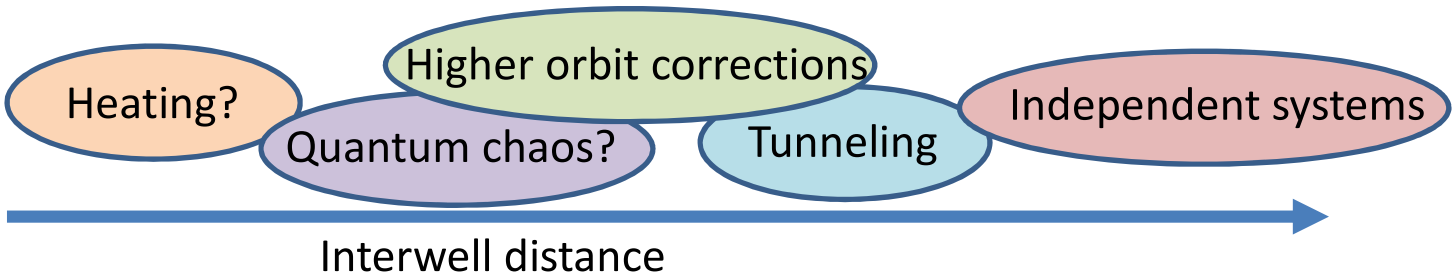
A word on micromotion

$$H(t) = \frac{p_i^2}{2m_i} + \frac{p_a^2}{2m_a} + \boxed{\frac{1}{8} m_i \Omega^2 r_i^2 \left(a + 2q\cos(\Omega t)\right)} + V_{dw}(r_a) - \frac{C_4}{(r_i - r_a)^4}$$

Paul trap

This problem can be solved with Floquet theory. The effect will be:

→ There is a smallest interwell distance beyond which the low-temperature Hamiltonian holds



See also: Nguyen et al., PRA 85, 052718 (2012).

Quantum analysis of Paul trap

$$H(t) = \frac{p_i^2}{2m_i} + \frac{1}{8} m_i \Omega^2 r_i^2 \left(a + 2q\cos(\Omega t)\right)$$

Secular frequency: $$\omega_i \approx \frac{\Omega}{2}\sqrt{a + \frac{q^2}{2}}.$$

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Ansatz for wavefunction: $\Psi(r_i, t) = \exp\left(-\frac{i}{4\hbar} m_i q \Omega^2 r_i^2 \sin(\Omega t)\right) w(r_i, t)$

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Results in a new Hamiltonian for *w(r$_i$,t)* $H_{eff} = H_{sec} + H_{mm}(t)$

$$H_{sec} = \frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 r_i^2$$ Secular Dynamics

$$H_{mm}(t) = -m_i g^2 \omega_i^2 r_i^2 \cos(2\Omega t) - g\omega_i \{r_i, p_i\} \sin(\Omega t)$$ MM Dynamics

Cook *et al.*, PRA **31**, 564 (1985).
Nguyen *et al.*, PRA **85**, 052718 (2012).

Double well with micromotion

- Expand in Floquet basis
- Diagonalise

Highly excited states
cross low lying tunneling states→
Avoided crossings in Floquet picture

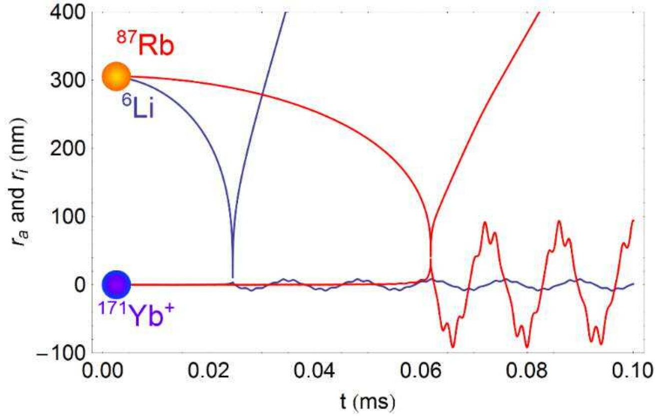
After doing the math:
Couplings scale with mass ratio m_a/m_i

Double well with micromotion

- Expand in Floquet basis
- Diagonalise

Highly excited states
cross low lying tunneling states→
Avoided crossings in Floquet picture

Classical picture:



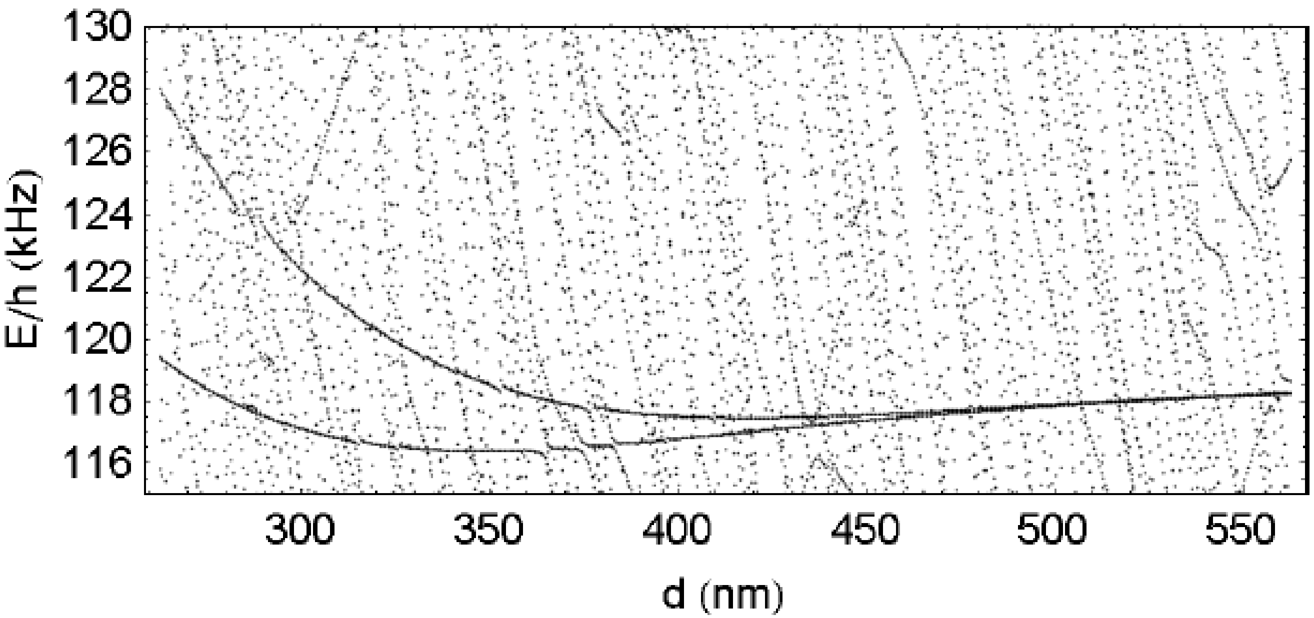
After doing the math:
Couplings scale with mass ratio m_a/m_i

	$^{171}Yb^+$ / ^{87}Rb	$^{171}Yb^+$ / ^{6}Li
m_i/m_a	1.97	28.42
S-wave limit	0.045μK	8.58μK
Rel. Heating	1	0.01

M. Cetina, A. T. Grier, V. Vuletić , *Phys. Rev. Lett.* , **109**,253201 (2012).

Example: Li and Yb^+

j = -2,..,2 (two echos only…)	~ 8000 basis states
^7Li and $^{171}Yb^+$	$\Omega = 2\pi$ 967 kHz
$\omega_i = 2\pi$ 137 kHz	$\omega_a = 2\pi$ 97 kHz
q = 0.4	



'Groundstate'

Small disturbance for lowest energy level

Conclusions (for now)

- Atom-ion systems show promise for quantum simulation.
- We derive state-dependent atom dynamics
- In the low temperature regime, the models follow those of solid state systems
- Experiment being building up now