Quantum frustration and quantum simulations with trapped ions and superconducting qubits

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IQIT – WP5: Patterns and challenges ahead

1) Long-term vision vs. medium-term applications

2) Integration and scalability: long way to go

3) Meanwhile, short to medium-term beyond iQIT:

3a) Quantum simulations/realizations

3b) Small-scale demonstrations

Feasible medium-term small-scale demonstrations:

1) Ground-state factorization

2) Quantum frustration

Many body system: $$\mathrm{H} = \sum_{P} h_P$$

Frustration is the impossibility to minimize simultaneously all the different "local" terms h_p

From classical to quantum frustration

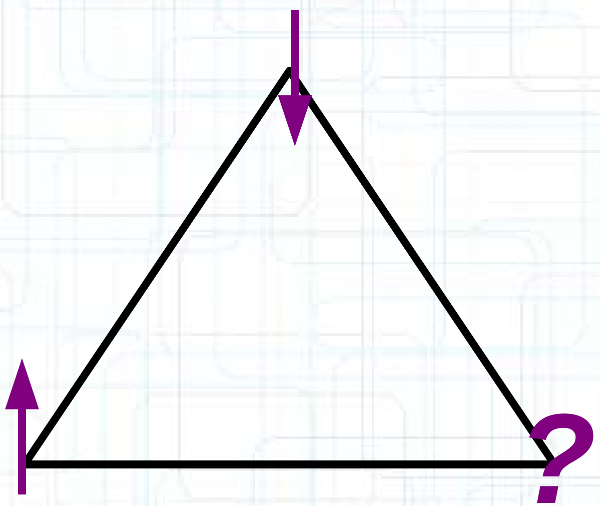
Many body system

either classical or quantum

$$H = \sum_P h_P$$

Geometric (classical) realization of the impossibility to minimize simultaneously all the different "local" terms h_p

Three anti-ferromagnetically interacting classical Ising spins on a triangle. Simulated with trapped ions by the Monroe group (Nature, 2010)



Sources of frustration, from classical to quantum

Classical World

Nontrivial geometry of the underlying physical space, e.g.: Heisenberg antiferromagnet on the 2-d triangular or Kagomé lattices, or on the 3-d pyrochlore lattice.

Competing interactions on different length scales, e.g. spin chains with ferromagnetic n.n. and antiferromagnetic n.n.n. Interactions.

Quantum World

Entanglement: Non-commutativity of the different local interaction terms.

Is there a clear, quantitative and observable characterization of quantum frustration? Amenable to quantum simulations? What consequences for the characterization/understanding of complex quantum matter?

Quantum frustration and entanglement

Classical Ising Hamiltonian

Three spins: all local terms commute

$$H=-J(S_1^z S_2^z+S_2^z S_3^z)$$

Each local term tends to align a pair of spins. All pairs are aligned in the global ground state. The ggs is a product state (unentangled).

Quantum XY Hamiltonian

Three spins: some local terms do not commute

$$H=-J[(S_1^z S_2^z+S_1^x S_2^x)+(S_2^z S_3^z+S_2^x S_3^x)]$$

Each local term tends to realize a Bell state of a spin pair but the second spin cannot be maximally entangled both with spin 1 and spin 3

Quantum frustration with no classical counterpart

Quantifying frustration

Infidelity between the “vacuum” local ground space of a spin pair and the “dressed” one, i.e. the reduced state of the same spin pair in the presence of the many-body interactions.

$$f_p = 1 - \mathrm{Tr}(\Pi \rho_p)$$

Π is the projector on the local ground space, ρ_p is the projection of the global ground state on the local Hilbert space

$$f_p \geq \epsilon_p^{(d)}$$

$$\epsilon_p^{(d)} = 1 - \sum_{k=1}^{d} \lambda_k^{\downarrow}$$

$\lambda_k^{\downarrow}$ are the eigenvalues of ρ_p in descending order

d is the dimension of the local Hilbert space of the spin pair

Frustration and Entanglement

$\epsilon_p^{(d)}$ → **Nielsen geometric entanglement monotone**

If d=1 $\epsilon_p^{(1)}$ → **Bi-separable Geometric Entanglement:**
Distance from the set of bi-separable states.

Classifying systems by the fundamental inequality: $f_p \geq \epsilon_p^{(d)}$

Frustration Free: $f_p = 0$

INES (INEquality Saturating) $f_p = \epsilon_p^{(d)}$

Non-INES $f_p > \epsilon_p^{(d)}$

Frustration hierarchies

$$f_p = 0$$

Frustration free: unfrustrated ===> vanishing entanglement. Necessary (but not sufficient) condition

$$f_p = \epsilon_p^{(d)}$$

INES: frustration <===> nonvanishing entanglement. Necessary and sufficient: *genuine quantum frustration*

$$f_p > \epsilon_p^{(d)}$$

Non-INES: Strict inequality ===> classical + quantum frustration

INES - NON INES transitions (I)

Working hypothesis I): quantum phase transitions in frustrated systems can be associated to INES – non INES transitions.

Working hypothesis II): the frustration-to-entanglement relations might allow to identify new classes of many-body effects beyond the “traditional” quantum phase transitions.

A paradigmatic case-study: the *XX*, *XXZ,* and Heisenberg Hamiltonians with competing antiferromagnetic interactions.

$$H = \sum_{ij} J_{ij} (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z)$$

$$J_{ij} > 0$$

INES - NON INES transitions (II): role of observable spin-spin correlations

For each pair (i,j) of interacting spins the local gs is a Bell singlet:

$$|g_{ij}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Taking into account all the symmetries of the Hamiltonian (Z_2 and conservation of the total magnetization), the frustration-to-entanglement relation takes the experimentally friendly form:

$$f_{ij} - \epsilon_{ij} = \max(0,\ 2xx_{ij},\ xx_{ij} + zz_{ij})$$

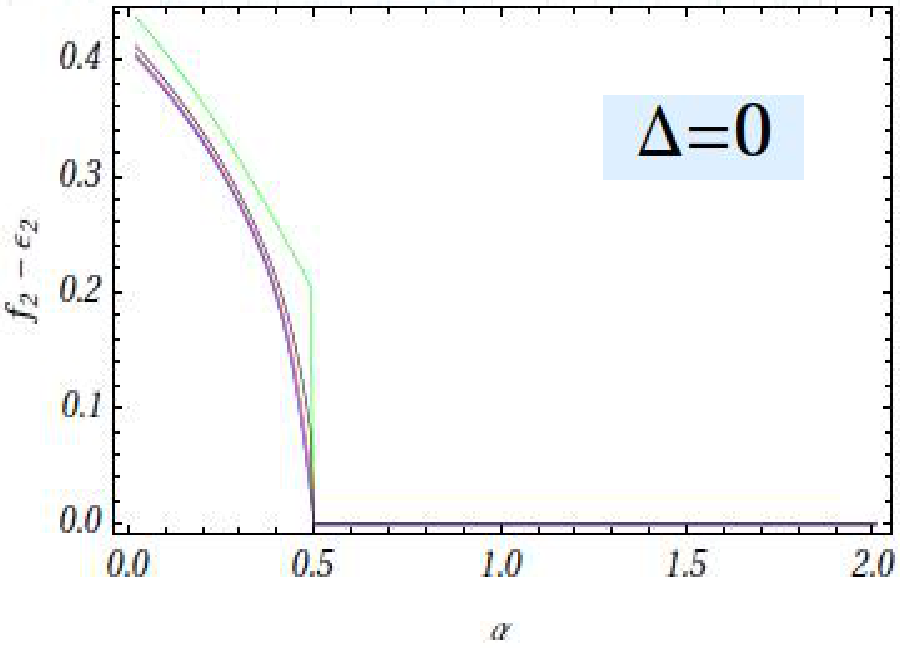
where xx_ij and zz_ij are short-hand for the spin-spin correlations respectively along the x and z direction.

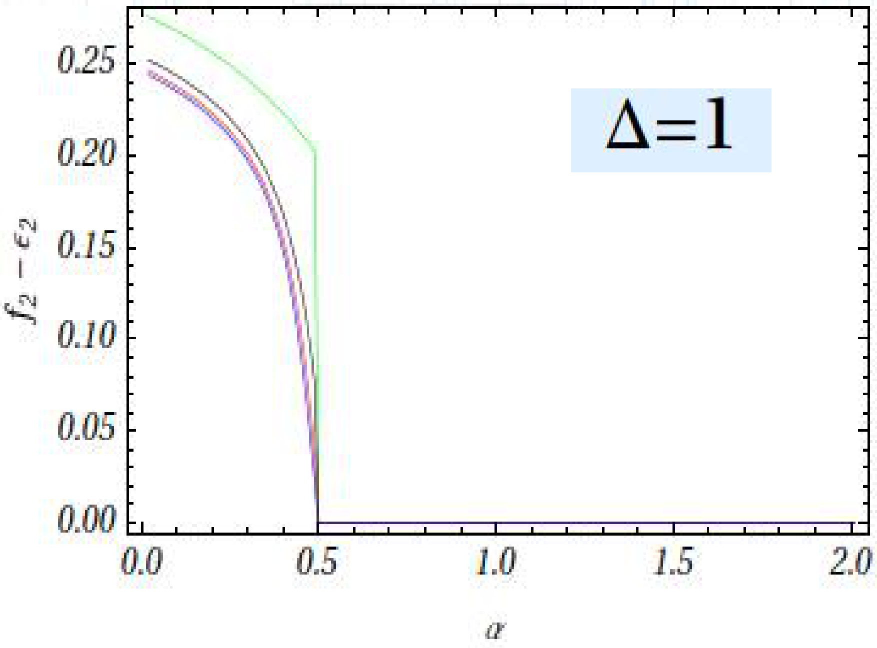
INES - NON INES transitions (III): origin of quantum dimerization

Simplest features and constraints:
I): 1-D system with pbc; II): Invariance under spatial translations;
III): All interactions are n.n. with uniform coupling J_1 > 0 and n.n.n with uniform coupling J_2 > 0.

Quantum parameter ruling possible qpts: ratio of the couplings.

$$\alpha = J_2 / J_1$$





α=0.5 : Majumdar-Ghosh point: transition to a dimerized gs

INES - NON INES transition (V)

Further observable characterization: from spin-spin correlations to static structure factor

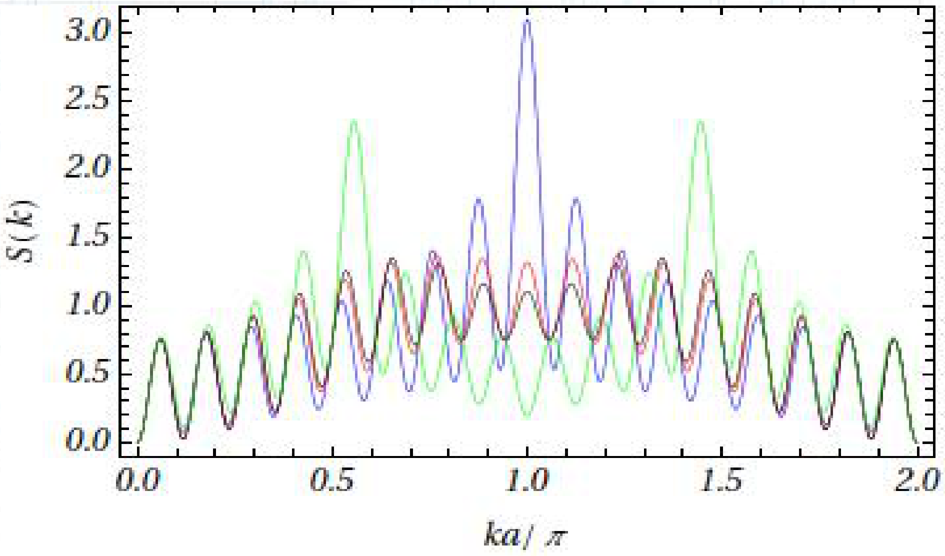
Static Structure Factor

$$S_f(\mathrm{k})=\frac{1}{N}\sum_{\mathrm{ij}}\mathrm{Exp}(-\mathrm{ik}\ (r_i-r_j))<S_i S_j>$$

Transition to local maxima crossing the dimerization point

$\Delta=1$

Heisenberg chain. N=18



Blue: α =0.15
Red: α =0.52
Black: α =0.57
Green: α =1.2

Quantum simulation of quantum frustration

Drawing experience from quantum simulation of LDE (WP1) so far:

Trapped ions - Design fairly hard. Readout fairly easy.

Outlook: XX Hamiltonian with J_1-J_2 coupling pattern seems to scale down the complexities in the design needed for the quantum simulation of LDE. Measurement and readout of *xx* spin-spin correlations?

Superconducting qubits: Design fairly easy. Readout fairly hard.

Outlook: XX Hamiltonian with J_1-J_2 coupling pattern seems to scale down the complexities in the measurement and readout of *xx* spin-spin correlations. Design of tunable n.n. and n.n.n. couplings?

References

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